

Reihenentwicklung

Lukas Grams

$$f(v) = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

$$f'(v) = -\frac{1}{2} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} \cdot \frac{-1}{c^2} \cdot 2v = \frac{v}{c^2} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} = \frac{v}{c^2 \sqrt{\left(1 - \frac{v^2}{c^2}\right)^3}}$$

$$f''(v) = \frac{1}{c^2} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} + \frac{v \cdot 3}{c^2 \cdot 2} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{5}{2}} \cdot \frac{-1}{c^2} \cdot 2v = \frac{1}{c^2} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} + \frac{3v^2}{c^4} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{5}{2}}$$

$$f'''(v) = \frac{-3}{2c^2} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{5}{2}} \cdot \left(-\frac{2v}{c^2}\right) + \frac{6v}{c^4} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{5}{2}} + \frac{3v^2 \cdot 5}{c^4 \cdot 2} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{7}{2}} \cdot \frac{-1}{c^2} \cdot 2v =$$
$$= \frac{3v}{c^4} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{5}{2}} + \frac{6v}{c^4} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{5}{2}} + \frac{15v^3}{c^6} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{7}{2}} =$$

$$f'''(v) = \frac{3v}{c^4} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{5}{2}} + \frac{15v^3}{c^6} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{7}{2}}$$

$$= \frac{9v}{c^4} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{5}{2}} + \frac{15v^3}{c^6} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{7}{2}}$$

$$f^{(4)}(v) = \frac{9}{c^4} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{5}{2}} + \frac{45v^2}{c^6} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{7}{2}} + \frac{45v^2}{c^6} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{7}{2}} + \frac{105v^4}{c^8} \left(1 - \frac{v^2}{c^2}\right)^{-\frac{9}{2}}$$

für $v=0$ einsetzen: $f(v)_T = f(0) + \frac{f'(0)}{1!} v + \frac{f''(0)}{2!} v^2 + \dots$

$$f(v)_T = 1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4}$$

nächste Seite bis $f^{(6)}(v)$