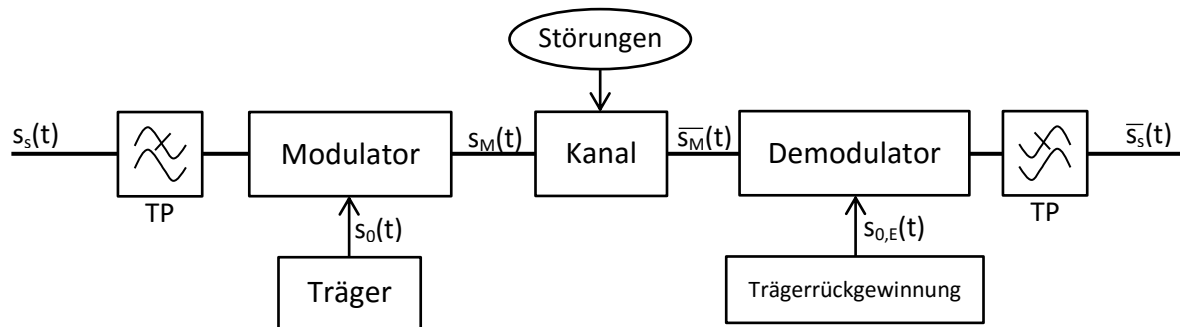


Modulation



$s_s(t)$ Basissignal

$s_0(t)$ Träger

$s_M(t)$ moduliertes Signal

$s_{0,E}(t)$ empfangsseitig zugesetzter Träger

$\bar{s}_s(t)$ demoduliertes Signal

Modulation ist die Veränderung von Signalparametern eines Trägers in Abhängigkeit eines zu modulierenden Signals. Die Demodulation ist die Rückgewinnung der in einer modulierten Trägerschwingung enthaltenen Signalinformation.

Gründe für Modulation:

- Mehrfachausnutzung des zur Verfügung stehenden Übertragungskanal
- Verbesserung der Signalanpassung an den Übertragungskanal
- Erzielung günstiger Störabstände

Modulation mit Sinusträger

Allgemeiner Sinusträger:

$$s(t) = \underbrace{\hat{s}(t)}_{\substack{\text{Amplitudenmodulation} \\ \text{AM}}} * \cos(\underbrace{\omega_0 t}_{\substack{\text{Trägerfrequenz} \\ \text{Frequenzmodulation} \\ \text{FM, PM}}} + \underbrace{\varphi(t)}_{\text{Phase}})$$

Amplitudenmodulation

$$s_M(t) = \hat{s}_M(t) * \cos(\omega_0 t + \varphi_0)$$

$$\hat{s}_M(t) = \hat{s}_0 + s_s(t)$$

Trägeramplitude (Offset)

$$\cos(\alpha + \beta) = \cos\alpha * \cos\beta - \sin\alpha * \sin\beta$$

$$s_M(t) = \underbrace{[\hat{s}_0 + s_s(t)] * \cos(\varphi_0) * \cos(\omega_0 t)}_{s_I(t)} - \underbrace{[\hat{s}_0 + s_s(t)] * \sin(\varphi_0) * \sin(\omega_0 t)}_{s_Q(t)}$$

Inphase-Komponente Quadratur-Komponente

$$s_M(t) = s_I(t) + s_Q(t)$$

Gewöhnliche Amplitudenmodulation: $\varphi = 0$, $\hat{s}_0 > 0$

AM: $s_M(t) = [\hat{s}_0 + s_s(t)] * \cos(\omega_0 t)$

Signal (reiner Sinus): $s_s(t) = \hat{s}_s * \cos(\omega_s t)$

$$s_M(t) = \hat{s}_0 * \cos(\omega_0 t) + \hat{s}_s * \cos(\omega_s t) * \cos(\omega_0 t)$$

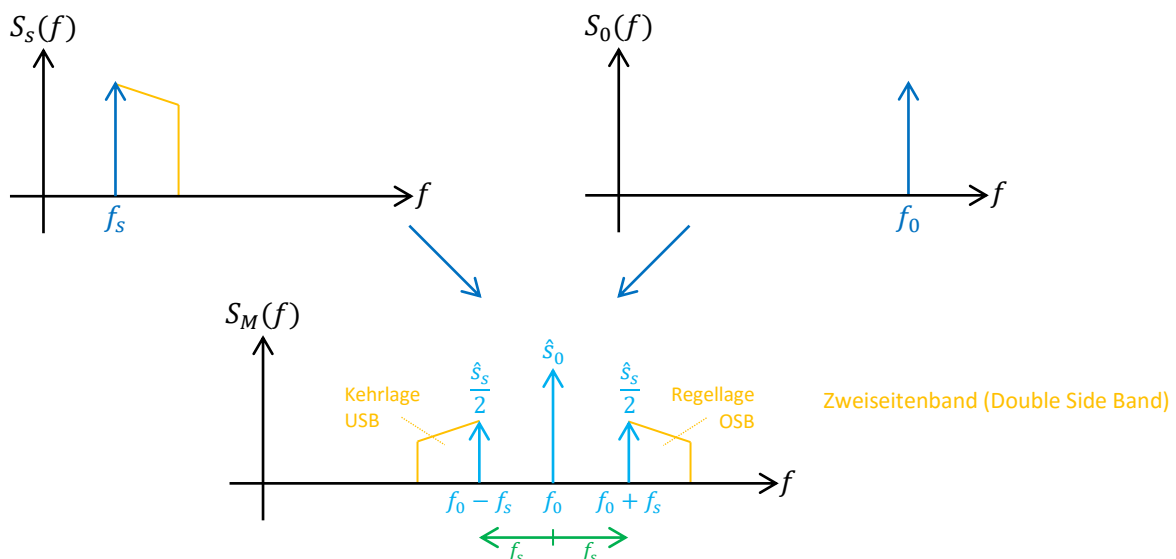
$$\cos\alpha * \cos\beta = \frac{1}{2} * [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

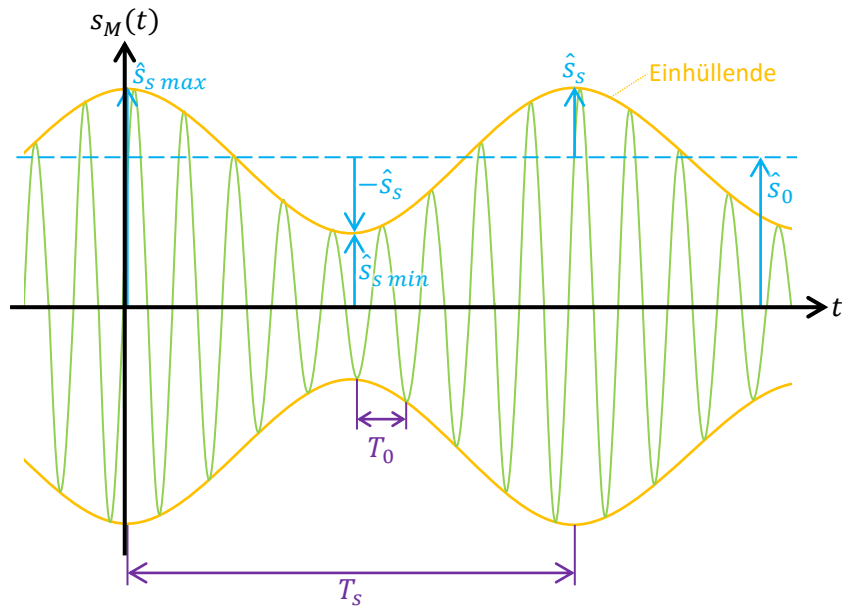
$$s_M(t) = \hat{s}_0 * \cos(\omega_0 t) + \frac{\hat{s}_s}{2} * \cos[(\omega_0 + \omega_s)t] + \frac{\hat{s}_s}{2} * \cos[(\omega_0 - \omega_s)t]$$

$$s_M(t) = \hat{s}_0 * \left\{ \cos(\omega_0 t) + \frac{m}{2} * \cos[(\omega_0 + \omega_s)t] + \frac{m}{2} * \cos[(\omega_0 - \omega_s)t] \right\}$$

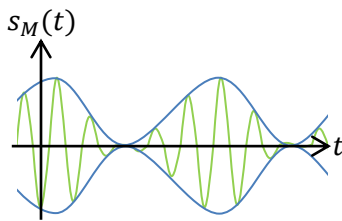
$$m = \frac{\hat{s}_s}{\hat{s}_0} \dots \text{Modulationsfaktor} = \frac{\text{Signalamplitude}}{\text{Trägeramplitude}}$$

Spektrum der Amplitudenmodulation:

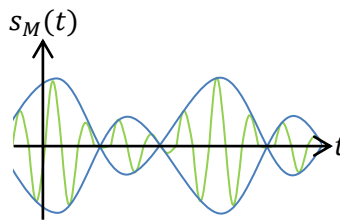


Zeitsignal:

m=1:



m>1:

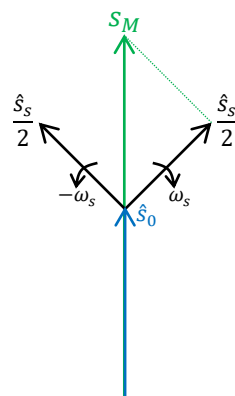


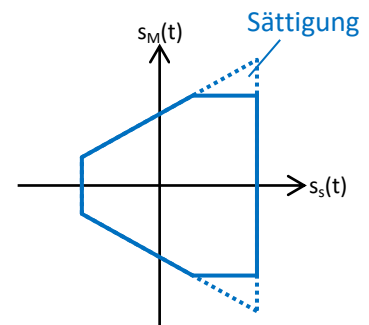
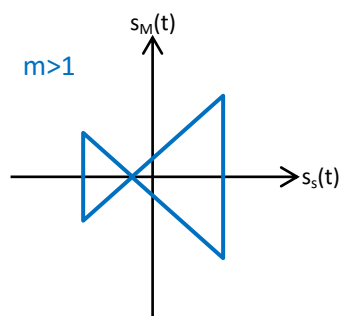
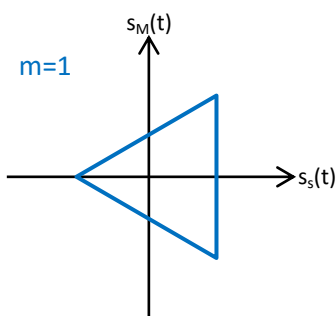
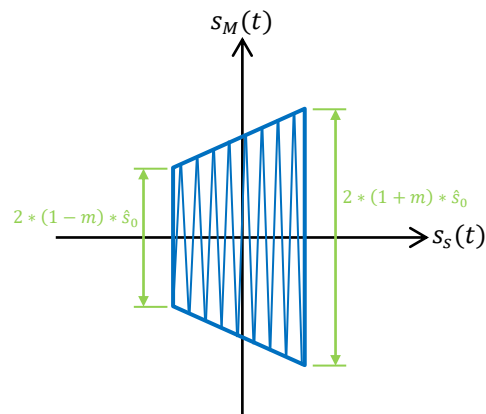
Bei Modulationsfaktor m>1 entstehen Oberwellen

komplexe Schreibweise, Zeigerdiagramm:

$$\underline{s}_M(t) = \underline{s}_T(t) * e^{j\omega_0 t}$$

$$\underline{s}_T(t) = \hat{s}_0 * \left(1 + \frac{m}{2} * e^{j\omega_s t} + \frac{m}{2} * e^{-j\omega_s t} \right)$$



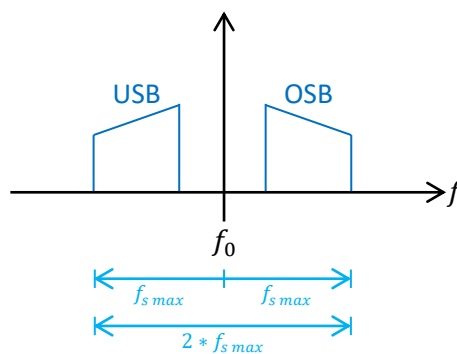
Modulationstrapez:**Leistungsinhalt:**

$$P_{ges} = P_0 + P_{OSB} + P_{USB} = P_0 * \left(1 + \frac{m^2}{2}\right)$$

$$m = 1: P_{ges} = P_0 * \left(1 + \frac{1}{2}\right) = 1,5 * P_0$$

Bandbreite:

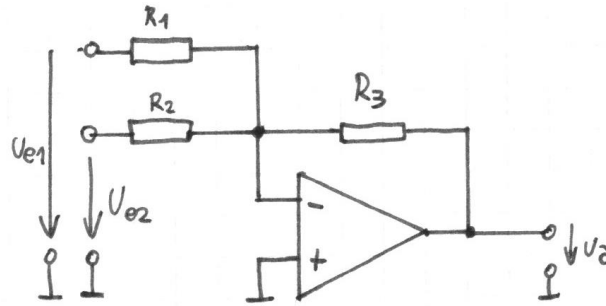
$$B_{HF} = 2 * B_{BB}$$



Modulationsschaltung:

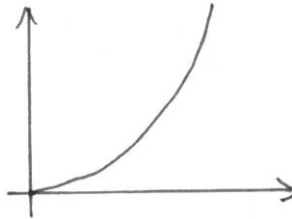
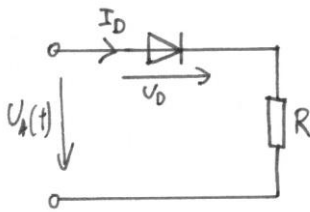
$$s_M(t) = [\hat{s}_0 + \hat{s}_s * \cos(\omega_s t)] * \cos(\omega_0 t)$$

Addition:



$$U_a = -R_3 * \left(\frac{U_{e1}}{R_1} + \frac{U_{e2}}{R_2} \right)$$

Multiplikator:



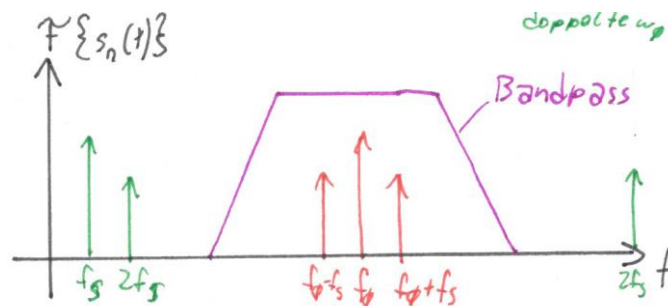
$$I_D = I_S * \left(e^{\frac{U_D}{m * U_T}} - 1 \right)$$

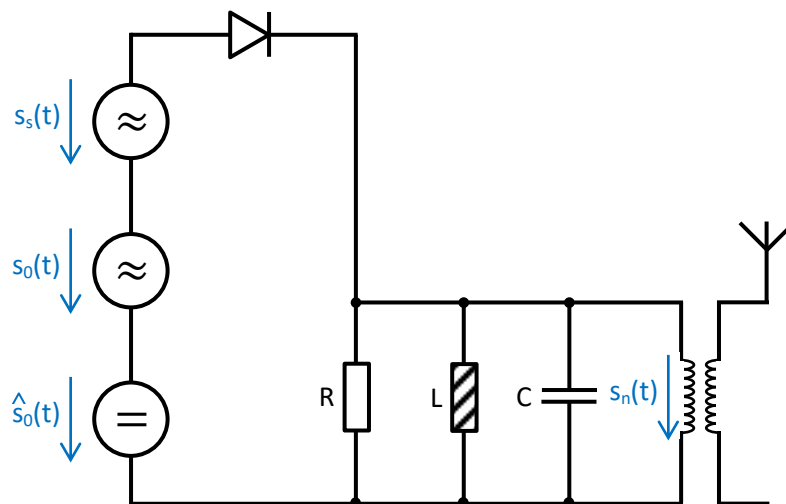
$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$U_D = U_A * [\cos(\omega_s t) + \cos(\omega_0 t)]$$

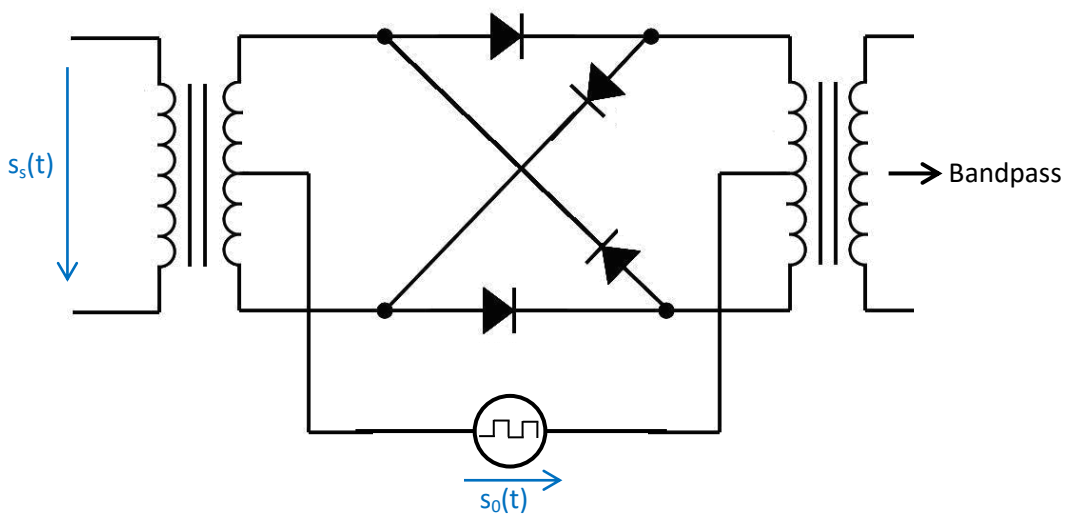
$$U_D^2 = U_A^2 * [\cos^2(\omega_s t) + 2 * \cos(\omega_s t) * \cos(\omega_0 t) + \cos^2(\omega_0 t)]$$

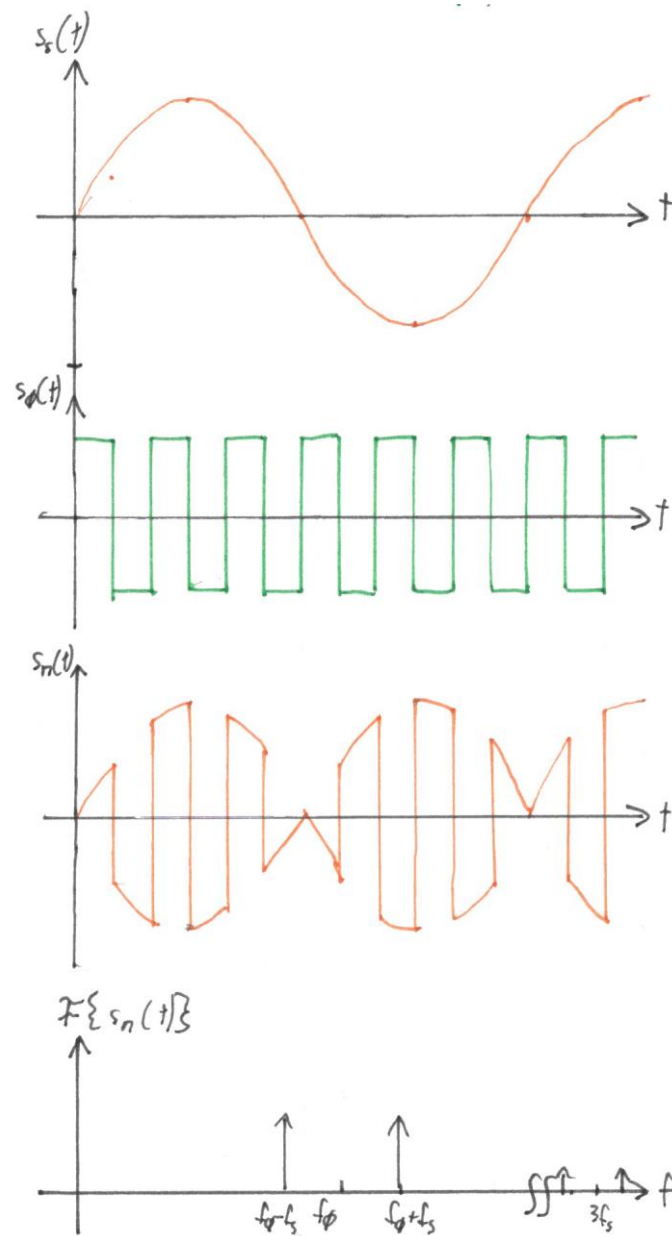
$$U_D^2 = U_A^2 * \left\{ \frac{1}{2} * [\cos(2\omega_s t) + 1] + \frac{2}{2} * [\cos((\omega_s + \omega_0) * t) + \cos((\omega_s - \omega_0) * t)] + \frac{1}{2} * [\cos(2\omega_0 t) + 1] \right\}$$



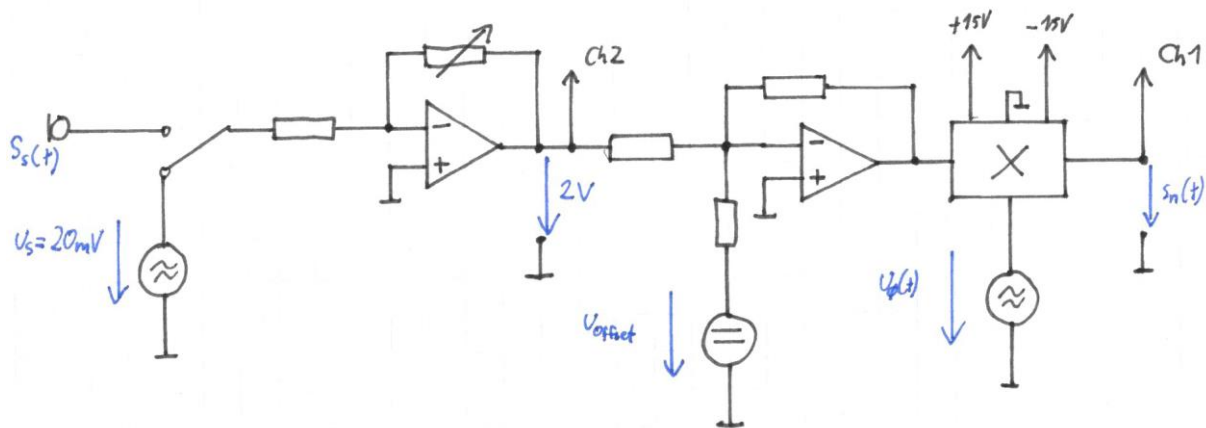
Diodenmodulator:**Ringmodulator:**

AM mit unterdrücktem Träger



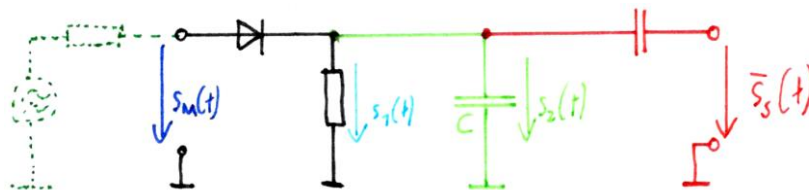


Amplitudenmodulator



Demodulator

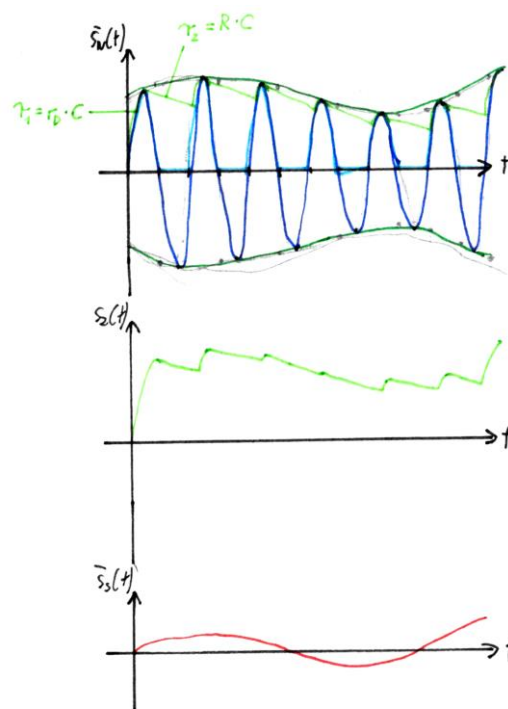
Hüllkurvendemodulator:

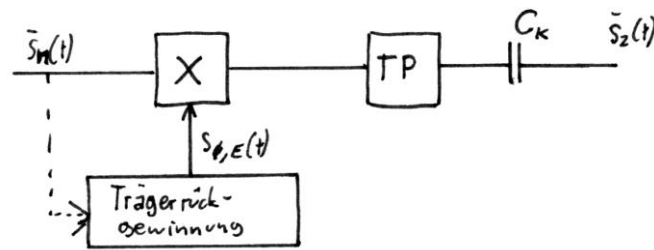


$$f_g = \frac{1}{2\pi \cdot R \cdot C}$$

$$f_s \ll f_g \ll f_0$$

$$\frac{10}{\omega_0} \leq R \cdot C \leq \frac{\sqrt{1-m^2}}{m \cdot \omega_{s \max}}$$



Köhrearter Demodulator:

$$s_n(t) = [\hat{s}_0 + s_s(t)] * \cos(\omega_0 t + \varphi_0)$$

$$s_{0,E}(t) = \hat{s}_{0,E} * \cos(\omega_{0,E} t + \varphi_{0,E})$$

$$s_n(t) * s_{0,E}(t) = [\hat{s}_0 + s_s(t)] * \hat{s}_{0,E} * \cos(\omega_0 t + \varphi_0) * \cos(\omega_{0,E} t + \varphi_{0,E})$$

$$s_n(t) * s_{0,E}(t) = [\hat{s}_0 + s_s(t)] * \hat{s}_{0,E} * \frac{1}{2} \left\{ \underbrace{\cos[(\omega_0 + \omega_{0,E}) * t + \varphi_0 + \varphi_{0,E}]}_{2 * \omega_0 \text{ mit TP wegfiltern}} + \underbrace{\cos[(\omega_0 - \omega_{0,E}) * t + \varphi_0 - \varphi_{0,E}]}_0 \right\}$$

Frequenzgleichheit: $\omega_0 = \omega_{0,E}$

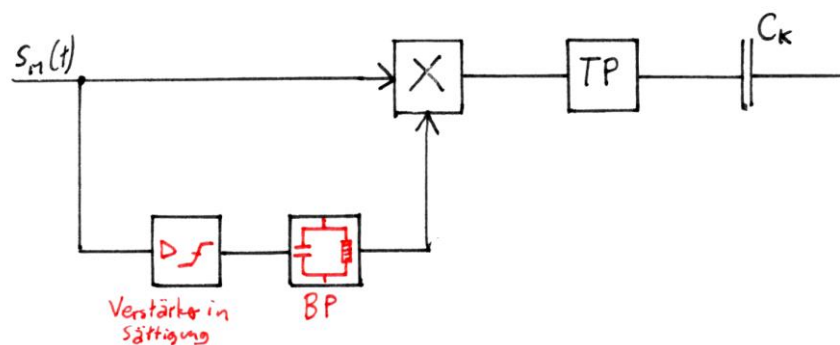
Nach TP: $[\hat{s}_0 + s_s(t)] * \frac{\hat{s}_{0,E}}{2} * \cos(\varphi_0 - \varphi_{0,E})$

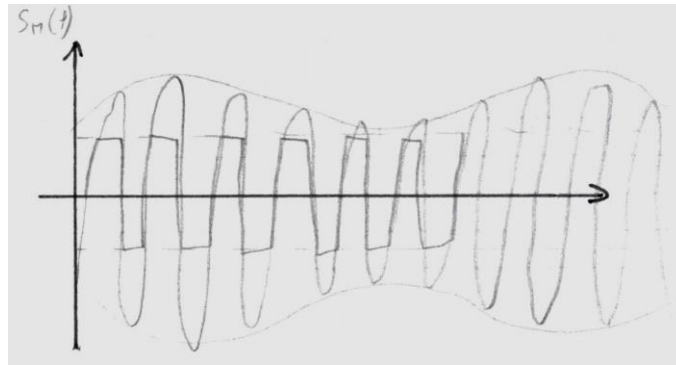
Nach C_k (Koppelkondensator):

$$s_s(t) * \frac{\hat{s}_{0,E}}{2} * \cos(\varphi_0 - \varphi_{0,E}) \rightarrow s_s(t)$$

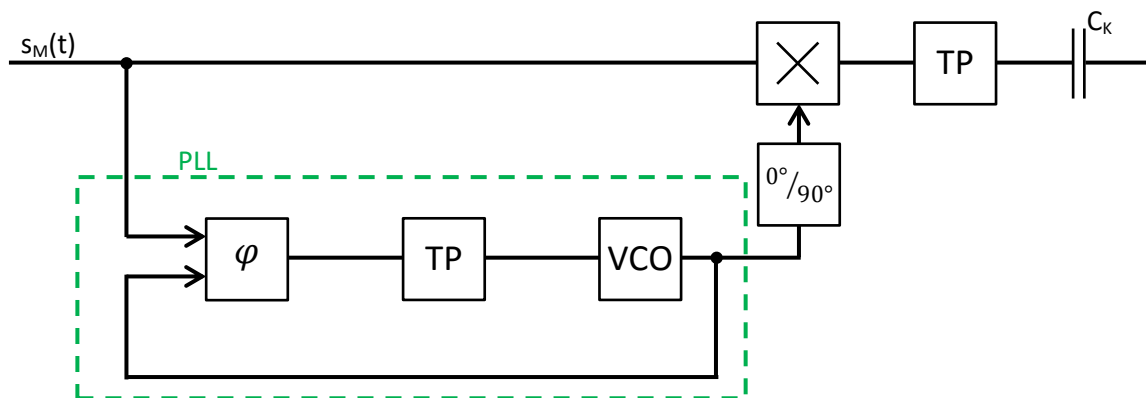
Phasengleichheit:
 $\varphi_0 = \varphi_{0,E}$

$\varphi_0 - \varphi_{0,E} = 90^\circ \rightarrow 0$ (kein Signal bei 90° Phasenverschiebung)

AM-Produktmodulator mit Trägerrückgewinnung:



Kohärenter Demodulator mit PLL (Phase-Locked-Loop / Phasenregelschleife):

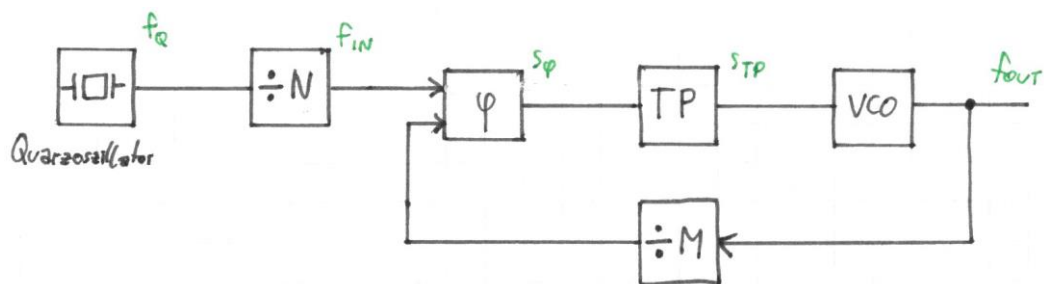


φ Phasendiskriminator

TP Schleifenfilter

VCO Voltage Controlled Oscillator

Takterzeugung mit PLL:



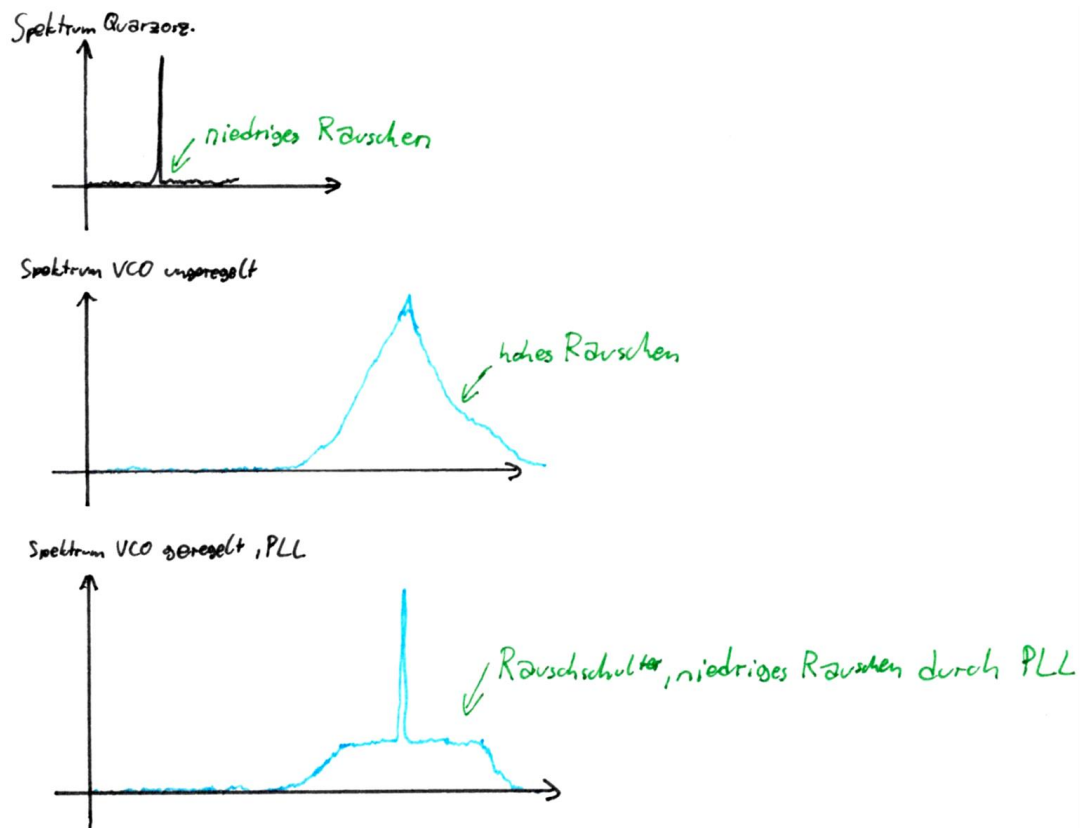
Quarzoszillator: Vorteil: hohe Güte, hohe Frequenzgenauigkeit

Nachteil: Frequenz nicht veränderbar

VCO, LC-Oszillator: Vorteil: Frequenz veränderbar

Nachteil: Frequenz ungenau

PLL



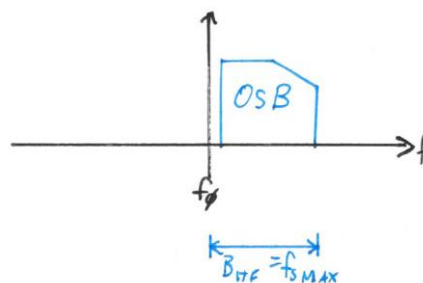
Einseiten-AM

(ESB-AM, SSB-AM, Single Side Band-AM)

$$s_m(t) = a_T * \hat{s}_o * \cos(\omega_0 * t) + \frac{\hat{s}_s}{2} * \cos((\omega_0 \pm \omega_s) * t)$$

a_T Trägerunterdrückung

-USB
+OSB

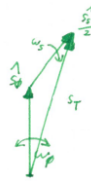


Vorteile der ESB-AM:

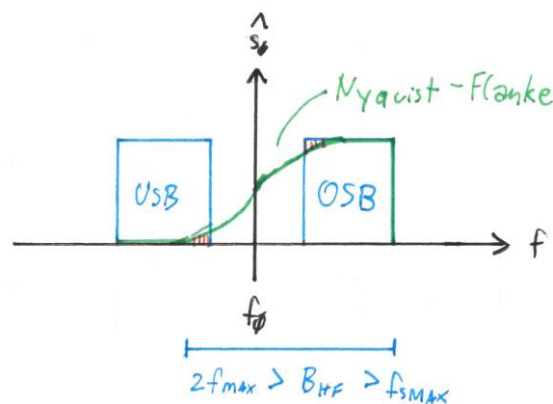
- Bessere Leistungsausnutzung
- Halbierung der erforderlichen Bandbreite
- Vergrößerter Rauschabstand bei gleicher Sendeleistung
- Geringere Empfindlichkeit gegen Mehrwegausbreitung (Selektiver Schwund)

Nachteile der ESB-AM:

- Größerer technischer Aufwand bei der Modulation & Demodulation
- Bei der Demodulation mit 2-Seitenband-AM-Demodulator ergeben sich nichtlineare Verzerrungen



Restseitenband-AM



Die Nyquist-Flanke ist Punktsymmetrisch zur Mittelfrequenz. Bei einem derartigen Flankenverlauf kann das Signal unverzerrt übertragen werden. Restseitenband-AM wird in der Fernsehtechnik für die Übertragung des Videosignals verwendet.

Frequenzmodulation

Momentanfrequenz

$$\omega(t) = \frac{d\psi_M(t)}{dt}$$

$$\psi_M(t) = \omega_0 t + \varphi_0(t)$$

$$\begin{aligned}\omega(t) &= \frac{d\psi_M(t)}{dt} \\ &= \omega_0 + \frac{d\varphi_0(t)}{dt} \\ &= \omega_0 + k s_S(t)\end{aligned}$$

Phase

$$\begin{aligned}\psi_M(t) &= \int_0^t \omega(\tau) d\tau + \psi_M(0) \\ &= \omega_0 t + k \int_0^t s_S(\tau) d\tau + \psi_M(0)\end{aligned}$$

Trägerschwingung

$$\begin{aligned}s_M(t) &= \hat{s}_0 \cos \psi_M(t) \\ &= \hat{s}_0 \cos \left[\omega_0 t + k \int_0^t s_S(\tau) d\tau + \psi_M(0) \right]\end{aligned}$$

Basisbandsignal

$$s_S(t) = \hat{s}_S \cos \omega_S t$$

Phase

$$\begin{aligned} \psi_M(t) &= \omega_0 t + k \int_0^t \hat{s}_S \cos \omega_S \tau \, d\tau \\ &= \omega_0 t + \frac{k \hat{s}_S}{\omega_S} \sin \omega_S t \end{aligned}$$

Frequenzhub

$$\Delta \omega = k \hat{s}_S$$

Modulationsindex

$$\eta = \frac{\Delta \omega}{\omega_S} = \frac{k \hat{s}_S}{\omega_S}$$

modulierte Trägerschwingung

$$s_M(t) = \hat{s}_0 \cos(\omega_0 t + \eta \sin \omega_S t)$$

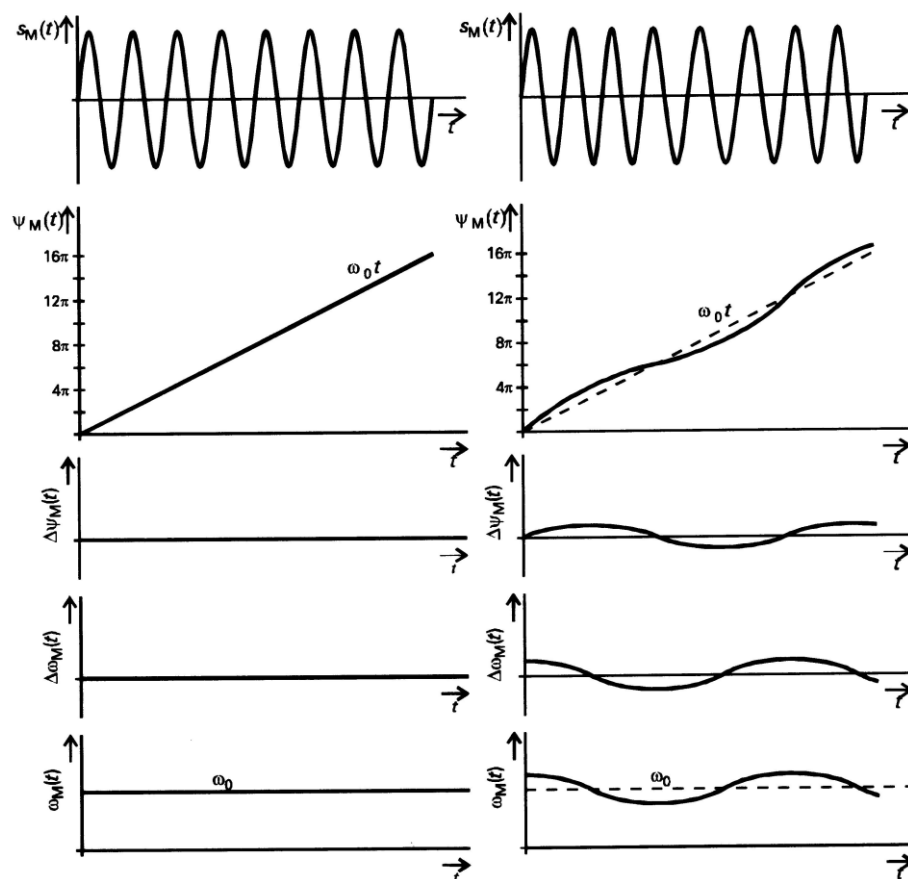


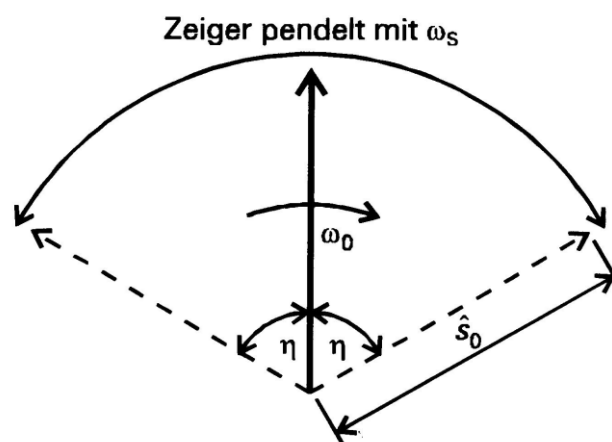
Abb. G.2-21 Zeitfunktionen des unmodulierten sowie des frequenzmodulierten Sinusträgers

Zeigerdiagramm der FM

$$\begin{aligned}s_M(t) &= \Re\left\{\hat{s}_0 e^{j\eta\sin\omega_s t} e^{j\omega_0 t}\right\} \\ &= \Re\left\{\underline{s}_T(t) e^{j\omega_0 t}\right\}\end{aligned}$$

komplexe Hüllkurve

$$\begin{aligned}\underline{s}_T(t) &= \underline{s}_M(t) e^{-j\omega_0 t} \\ &= \hat{s}_0 e^{j\eta\sin\omega_s t}\end{aligned}$$



Spektrum der FM

$$\begin{aligned}s_M(t) &= \hat{s}_0 \Re\left\{e^{j\omega_0 t} \sum_{-\infty}^{+\infty} J_n(\eta) e^{j\omega_s t}\right\} \\ &= \hat{s}_0 \sum_{-\infty}^{+\infty} J_n(\eta) \cos[(\omega_0 + n\omega_s)t] \\ &= \hat{s}_0 \{J_0(\eta) \cos \omega_0 t + \\ &\quad + J_1(\eta) \cos[(\omega_0 + \omega_s)t] + J_{-1}(\eta) \cos[(\omega_0 - \omega_s)t] + \\ &\quad + J_2(\eta) \cos[(\omega_0 + 2\omega_s)t] + J_{-2}(\eta) \cos[(\omega_0 - 2\omega_s)t] + \\ &\quad + J_3(\eta) \cos[(\omega_0 + 3\omega_s)t] + J_{-3}(\eta) \cos[(\omega_0 - 3\omega_s)t] + \\ &\quad + \dots\}\end{aligned}$$

Bessel-Funktion n -ter Ordnung $J_n(\eta)$.

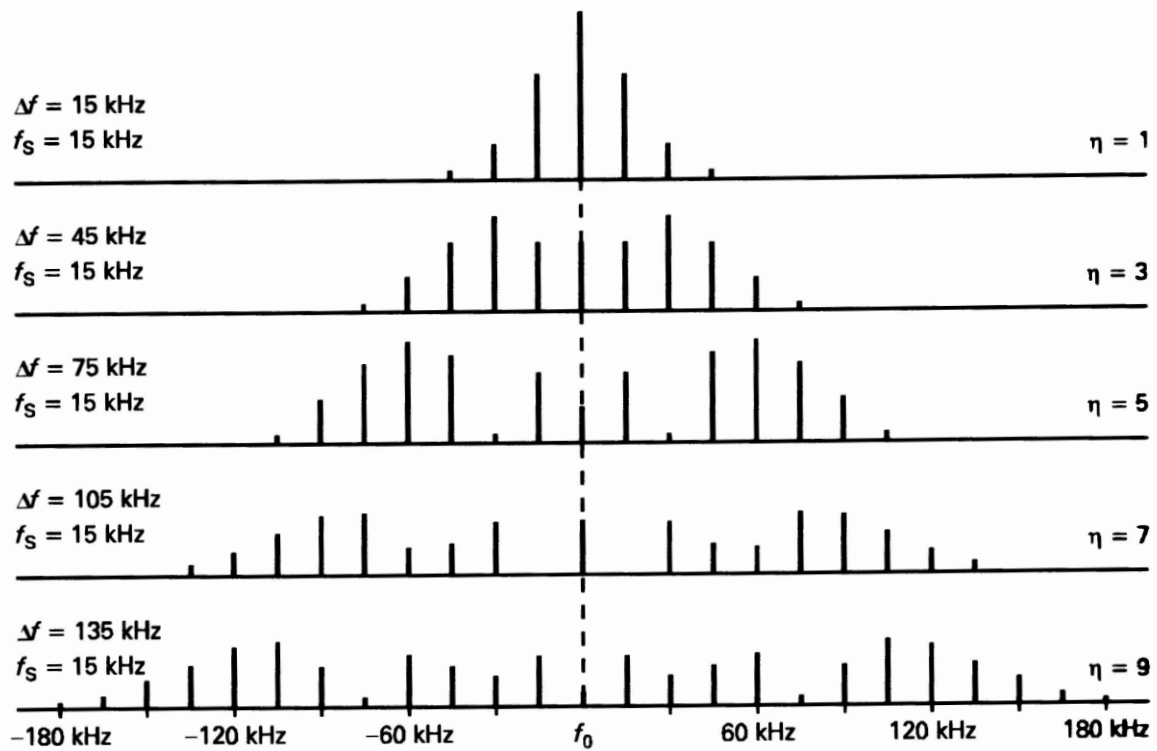


Abb. G.2-23 Betragsspektren einer frequenzmodulierten Schwingung bei Variierung des Modulationsindex und konstanter Signalfrequenz

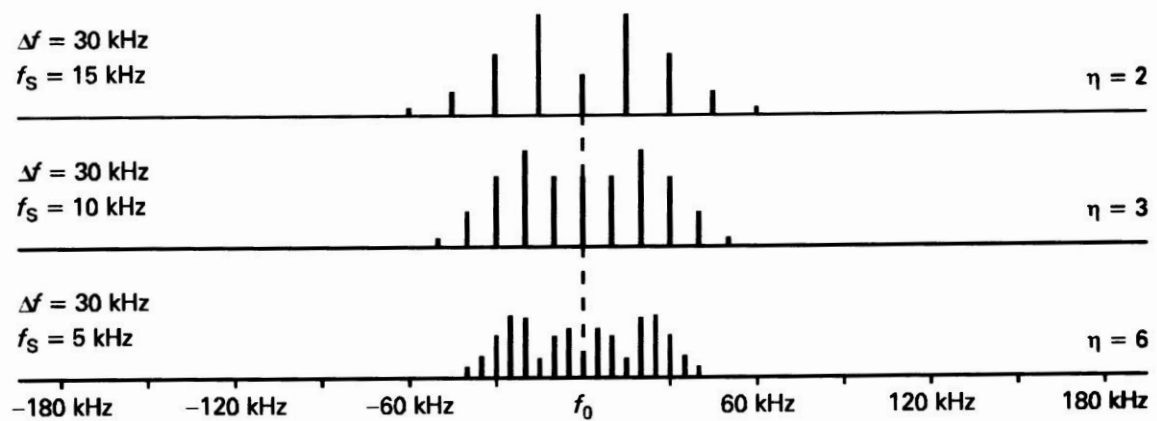


Abb. G.2-24 Betragsspektren einer frequenzmodulierten Schwingung bei Variierung des Modulationsindex und konstantem Hub

Bandbreite der FM

Schmalband FM: $\eta \leq 1$

$$B_{HF} = 2 B_{BB}$$

Breitband FM: $\eta > 1$

$$B_{HF} \approx 2(\eta + 1)B_{BB}$$

System	Trägerfrequenz f_0	Frequenzhub $\Delta f = \Delta\omega/2\pi$	Basisband	Mod.-Index η
Rundfunk (UKW)	(87,5 ... 108) MHz	± 75 kHz	30 Hz ... 15 kHz	5 ... 2500
TV-Richtfunk	70 MHz (ZF)	± 4 MHz	30 Hz ... 5 MHz	0,8 ... 133333

Tab. G.2-2 Typische Parameter von frequenzmodulierten Systemen

UKW:

$$f_s = 30 \text{ kHz}$$

$$\Delta f = 75 \text{ kHz} \quad \eta = \frac{75 \text{ kHz}}{30 \text{ kHz}} = 2500$$

$$B_{HF} = 2(\eta + 1) \cdot B_{BB} = 2 \cdot 2501 \cdot 30 \text{ kHz} = \underline{\underline{150 \text{ kHz}}}$$

$$f_s = 15 \text{ kHz}$$

$$\Delta f = 75 \text{ kHz} \quad \eta = \frac{75 \text{ kHz}}{15 \text{ kHz}} = 5$$

$$B_{HF} = 2 \cdot 6 \cdot 15 \text{ kHz} = \underline{\underline{180 \text{ kHz}}}$$

Kanalabstand UKW: 300 kHz

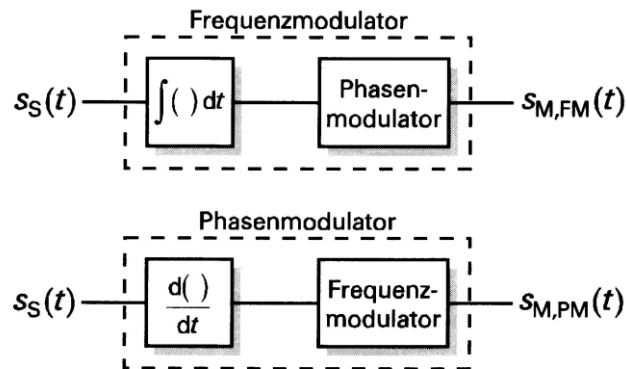
Phasenmodulation

FM

$$\begin{aligned}\omega(t) &= \frac{d\psi_M(t)}{dt} \\ &= \omega_0 + \frac{d\varphi_0(t)}{dt} \\ &= \omega_0 + k s_S(t)\end{aligned}$$

PM

$$\begin{aligned}\varphi_0(t) &= k s_S(t) \\ \omega(t) &= \frac{d\psi_M(t)}{dt} \\ &= \omega_0 + \frac{d\varphi_0(t)}{dt} \\ &= \omega_0 + k \frac{ds_S(t)}{dt}\end{aligned}$$



Preemphasis und Deemphasis:

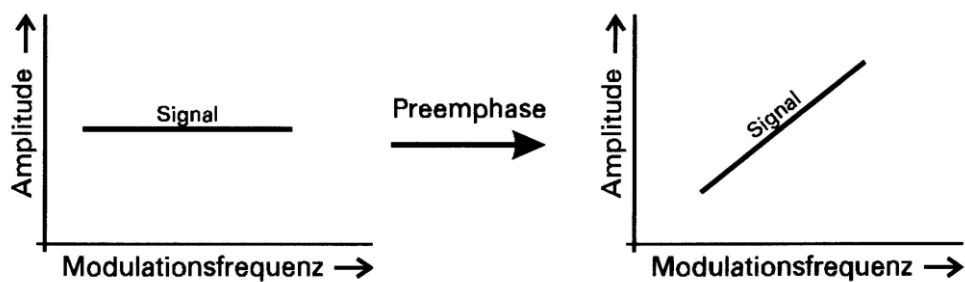


Abb. G.2-34 Preemphasis beim Sender

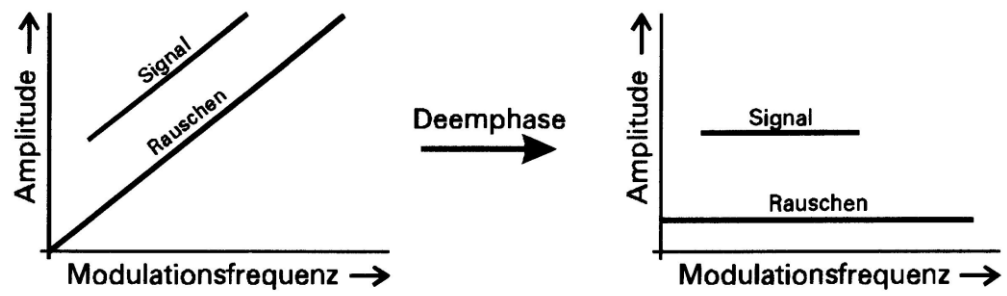
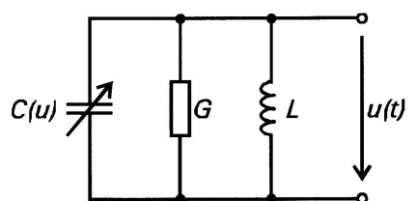
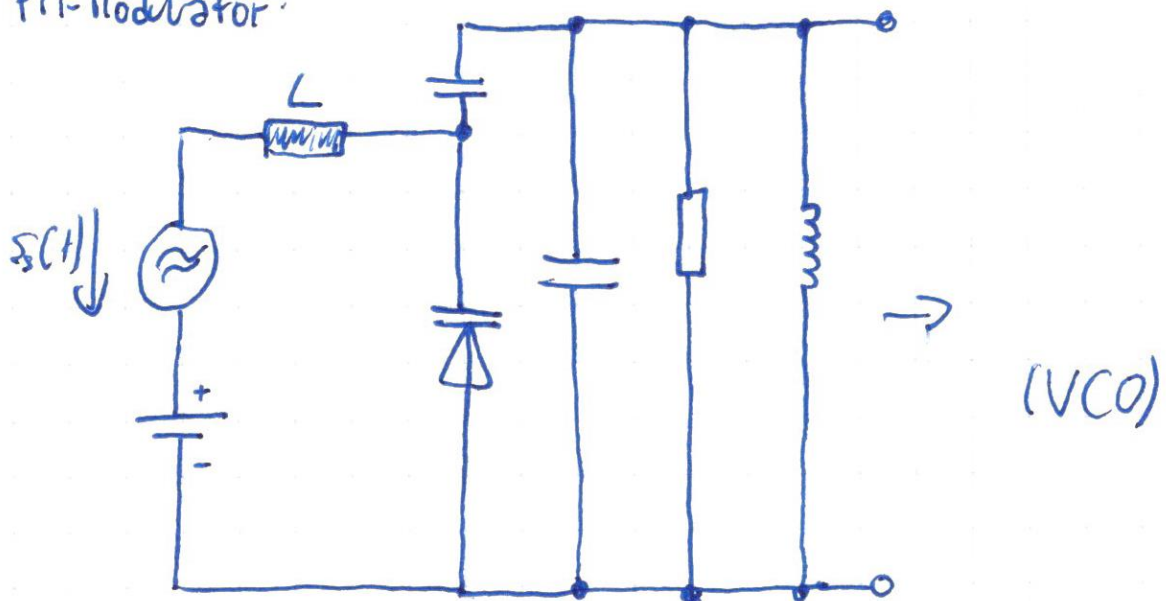


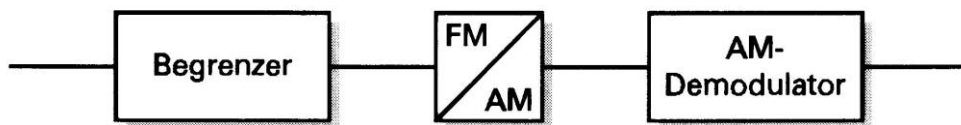
Abb. G.2-35 Deemphase beim Empfänger

FM - Modulator

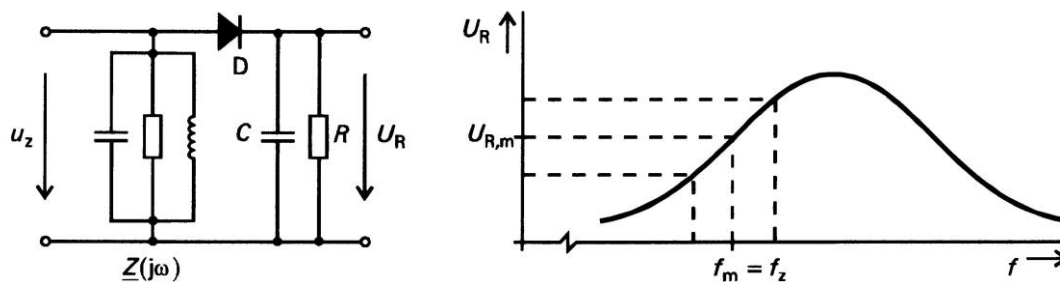
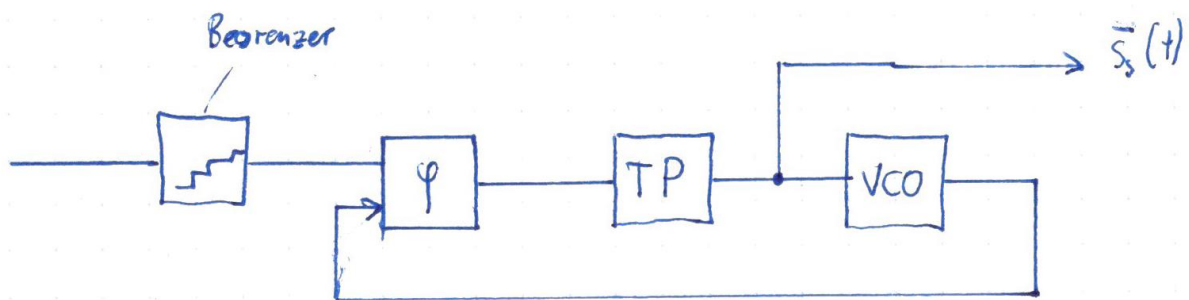
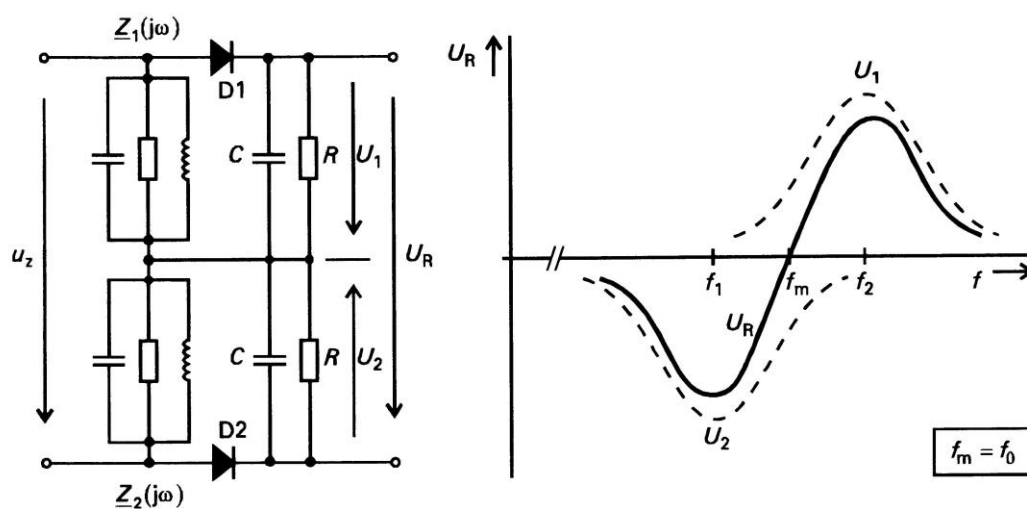


FM-Modulator:

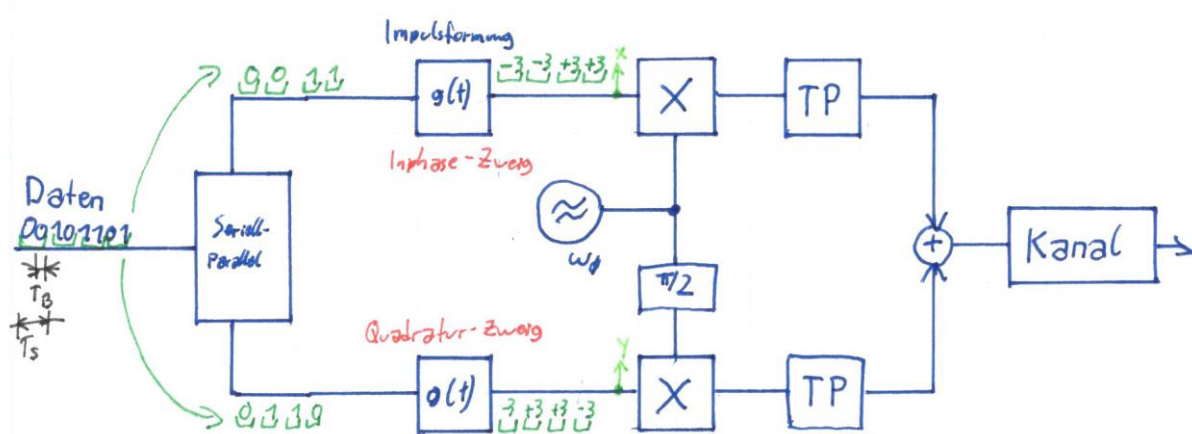


FM - Demodulator**Abb. G.2-37** FM-Demodulation

FM-Demodulator mit PLL

**Abb. G.2-38** Der Eintakt-Flankendemodulator**Abb. G.2-39** Der Gegentakt-Flankendemodulator

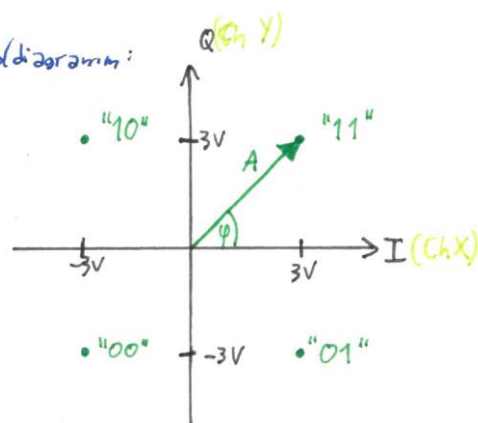
Digitale Modulation



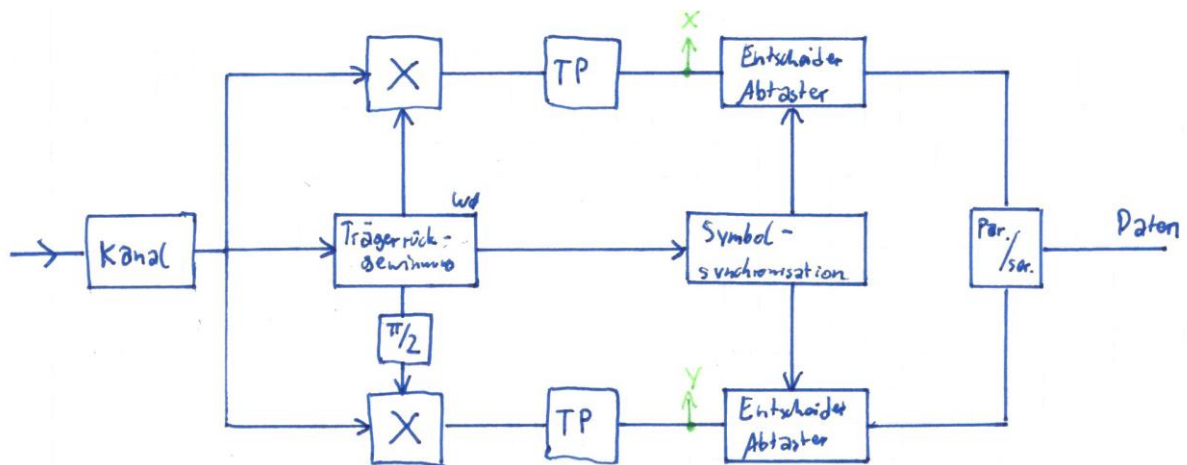
T_B Bitdauer

T_s Symboldauer

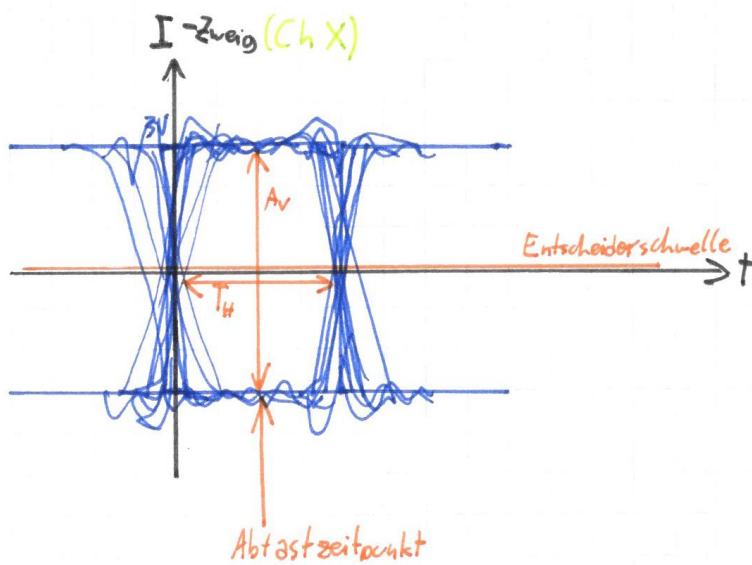
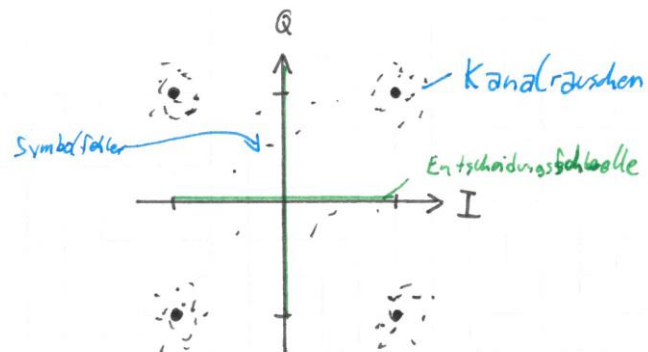
Zustandsdiagramm, Symboldiagramm:



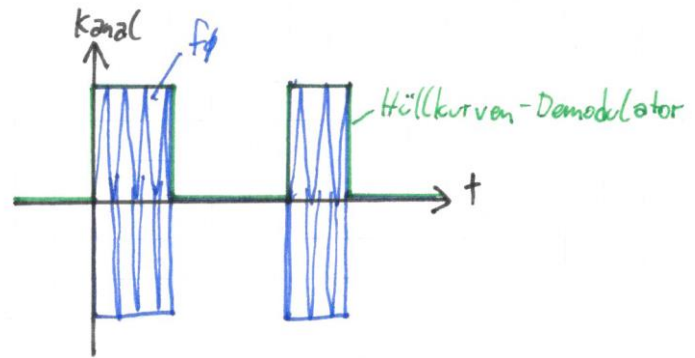
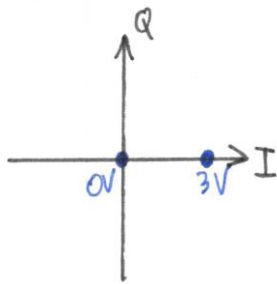
A... Abstand zum Ursprung, entspricht der Leistung die übertragen werden muss.



Zustandsdiagramm nach Demodulator:

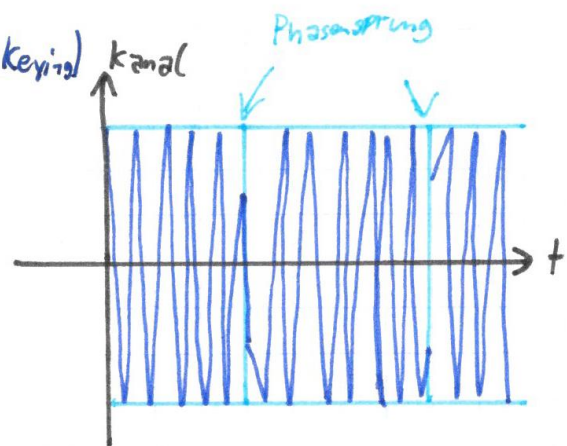
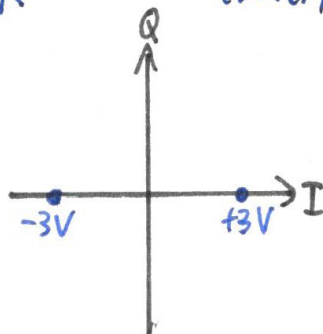


ONOFF - keying

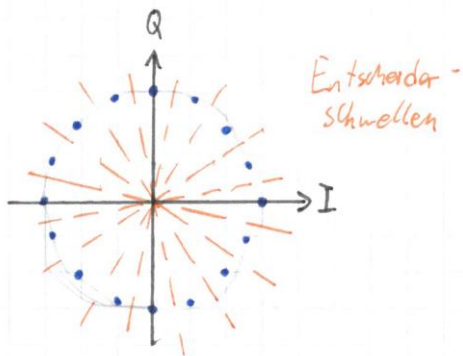


BPSK

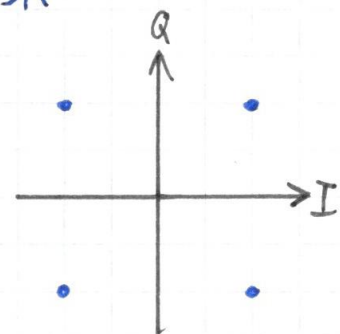
(Binary Phase Shift Keying)



16-PSK

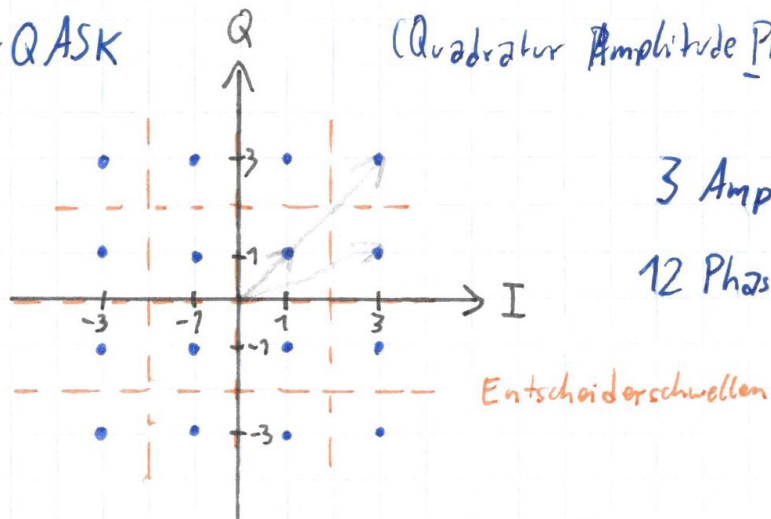


QASK



16-QASK

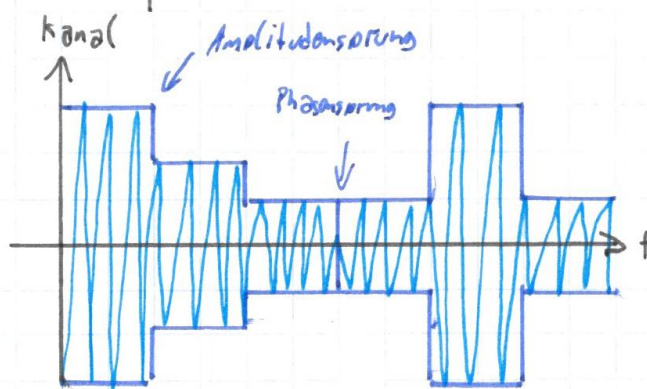
(Quadratur Amplitude Phase Shifted Keying)



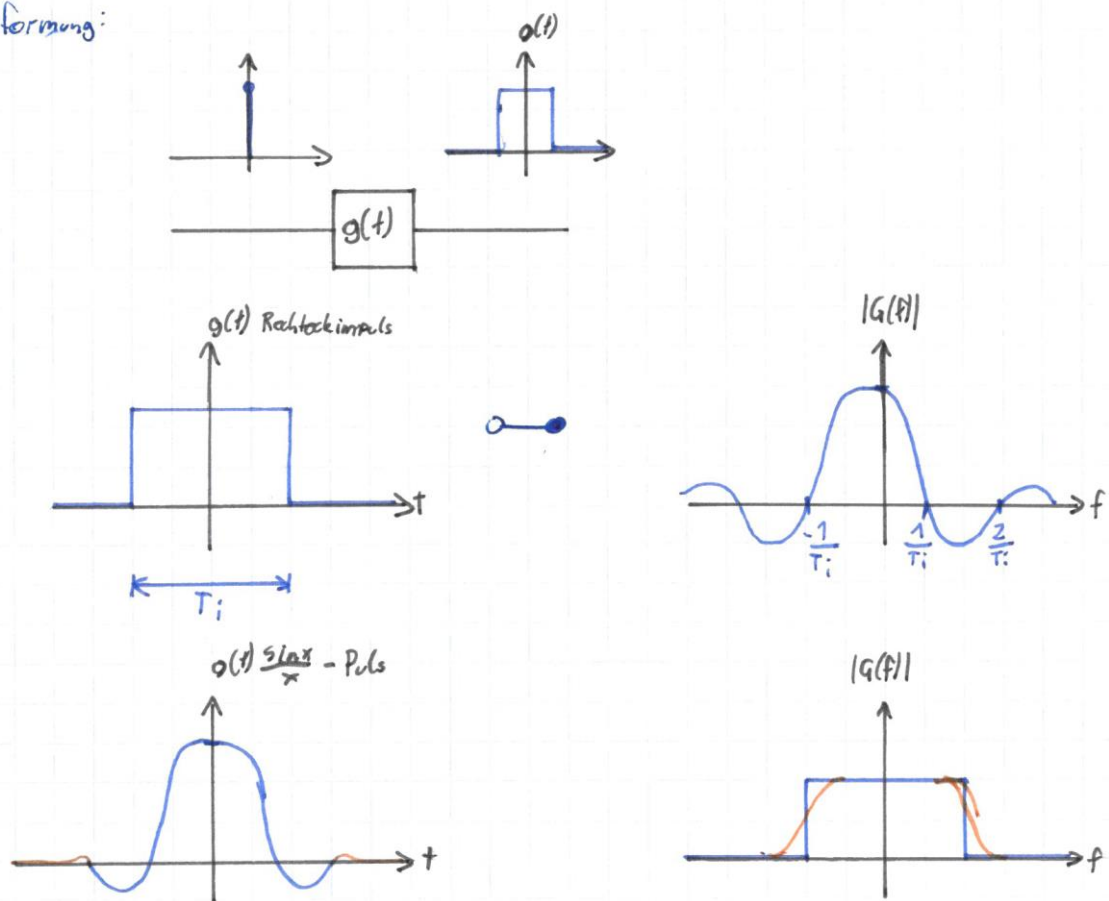
3 Amplitudenzustände

12 Phasenzustände

Entscheidungsschwellen



Impulsformung:



Pulse mit ISI (Intersymbol-Interferenz)

